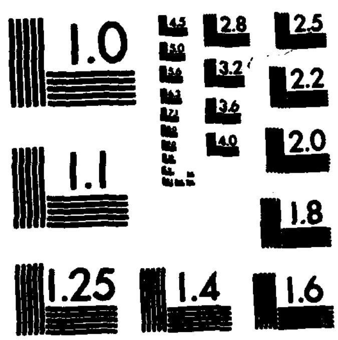
AD-A124 830 REDUCTION OF ALL-POLE PARAMETER ESTIMATOR BIAS BY SUCCESSIVE AUTOCORRELATION(U) RICE UNIV HOUSTON TEX DEPT OF ELECTRICAL ENGINEERING D MCGINN ET AL. DEC 82 1/1

UNCLASSIFIED TR-8221 N00014-81-K-0565 F/G 12/1 NL

END
FILMED
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

AD A124 830

(12)

RICE UNIVERSITY

GEORGE R. BROWN SCHOOL OF ENGINEERING

DTIC
ELECTE
FEB 23 1983
S D
A

DEPARTMENT OF ELECTRICAL ENGINEERING

HOUSTON, TEXAS

DTIC FILE COPY

This document has been approved for public release and sale; its distribution is unlimited.

83 02 023 164

REDUCTION OF ALL-POLE PARAMETER ESTIMATOR BIAS BY SUCCESSIVE AUTOCORRELATION

Darcy McGinn and Don H. Johnson
Department of Electrical Engineering
Rice University
Houston, Texas 77251

Technical Report #8221

This work was sponsored by the Office of Naval Research under Contract N00014-81-K-0565.

REDUCTION OF ALL-POLE PARAMETER ESTIMATOR BIAS BY SUCCESSIVE AUTOCORRELATION

Darcy McGinn and Don H. Johnson

Abstract

Conventional all-pole parameter estimators applied to noise corrupted all-pole sequences result in biased estimates. This paper describes a procedure by which reduction of this bias is accomplished by applying pole-preserving, signal to noise ratio improving functions to the sequence. Correlation like pole-preserving functions are investigated and pole dependent signal to noise ratio improvement is described. An all-pole parameter estimator using successive application of a pole-preserving *function* (successive autocorrelation) is given. Comparison is made with the least squares combination of the higher order Yule-Walker equations, an approach to bias reduction reported by Cadzow. Successive autocorrelation is found to result in improved performance, with estimates of higher Q poles being most effectively enhanced.

Introduction

Goldberger [1] and others have shown that when the classic least squares estimator of all-pole parameters (Yule-Walker equations) is applied to noise corrupted all-pole sequences, the estimates are biased. To avoid biased estimators, Chan [2] applied the method of instrumental variables to all-pole spectrum estimation. The use of lagged data as the instrumental variables results in the higher order Yule-Walker equations (HOYWE) investigated by Kay [3] and Gingras [4]. Kay noted that the large variance of estimators based on the HOYWE was a serious drawback to the approach. This problem was addressed by Cadzow [5] by forming a least squares combination of the overdetermined system of HOYWE. The elements of the final normal equations contain

fourth powers of the data, and can be thought of as correlations of the correlation estimates. Insight into these approaches, and a generalization of the algorithms used may be attained by viewing the elements of the HOYWE as a data sequence, and the autocorrelation estimator as a pole-preserving, signal to noise ratio improving function.

Problem Definition

Let s_t be a pth order all-pole sequence given by the difference equation

$$s_t = \sum_{i=1}^{p} a_i s_{t-i} + G\delta_t \tag{1}$$

The noise corrupted sequence is given by

$$x_t = s_t + \varepsilon_t \qquad t=0,\ldots,N-1 \tag{2}$$

where ε_t is a: identically distributed zero mean noise sequence with variance σ^2. The all-pole sequence $\{s_t\}$ is deterministic and represents the impulse response of an all-pole digital system. This model is often used in impulse response analysis, speech processing and the analysis of transients.

Pole-Preserving Functions

A function of an all-pole sequence is pole-preserving if the result of applying the function to an all-pole sequence is a sequence with the same poles as the original. For example, the autocorrelation of $\{s_t\}$ is a pole-preserving function. This becomes evident by expanding the z-transform of the autocorrelation for lags ≥ 0. The z-transform $S(z)$ of a pth order all-pole sequence is given by

$$S(z) = \frac{G}{\prod_{i=1}^{p}(1-a_i z^{-1})} = \sum_{i=1}^{p} \frac{Gb_i}{1-a_i z^{-1}} \tag{3}$$

The z-transform of the autocorrelation of $\{s_t\}$ is given by

$$Z\{R_s(n)\} = S(z)S^*(z^{-1})$$
$$= G^2 \sum_{i=1}^{p} \frac{c_i}{1-a_i z^{-1}} + G^2 \sum_{j=1}^{p} \frac{c_j^*}{1-a_j^* z}$$

The c_i are complicated functions of the b_i and the a_i. The terms in z^{-1} represent the sequence $\{R_s(n)\}$ for $n \geq 0$. The z-transform of this right sided sequence is given by

$$Z\{R_s^+(n)\} = G^2 \sum_{i=1}^{p} \frac{c_i}{1-a_i z^{-1}} \tag{4}$$

Comparing (4) with (3) one notes that $\{R_s^+(n)\}$ has the same poles as $\{s_t\}$, but the modes have different relative amplitudes. Stated another way, $\{R_s^+(n)\}$ has a z-transform having both poles and zeroes; the zeroes are due to the modified values of the modes. For lags of p or greater, $R_s^+(n)$ obeys the same difference equation as the original sequence.

Finite Length Pole-Preserving Functions

All practical pole-preserving functions will operate with finite data lengths. One function often used to form the Yule-Walker equations is the biased autocorrelation estimate given by

$$R_s(n) = \sum_{t=0}^{N-n-1} s_t s_{t+n} \qquad n = 0, 1, \ldots, N-1 \tag{5}$$

Application to a noiseless all-pole sequence (1) results in

$$R_s(n) = \sum_{i=1}^{p} a_i R_s(n-i) - \sum_{i=1}^{p} a_i \sum_{t=N-n}^{N-n+i} s_t s_{t+n-i} + s_0 G \delta_n$$

The function has preserved the poles, but the second term represents an equation error which produces parameter estimator errors even in the noiseless case.

Another common choice of correlation estimator is the "covariance" type.

$$K_s(n) = \sum_{t=0}^{M-1} s_t s_{t+n} \qquad n=0,1,\ldots,N-M \tag{6}$$

where M is the number of terms in the averaging computation. Application to (1) results in

$$K_s(n) = \sum_{i=1}^{p} a_i K_s(n-i) + s_0 G \delta_n$$

Equation (6) is pole-preserving, and the sequence $\{K_s(n)\}$ obeys the same difference equation as $\{s_t\}$ for $p \le n \le N-M$. Using the elements of $\{K_s(n)\}$ in the Yule-Walker equations would add no error to the parameter estimates.

A number of observations about (6): 1) The number of elements in $\{K(n)\}$ is less than or equal to the original sequence, 2) The property of satisfying (1) for all n relies only on the constant summation limits, 3)The s_t in (6) may be replaced by any length M sequence $\{v_t\}$ which is not orthogonal to $\{s_t\}$ without affecting the pole-preserving property. A more general form for covariance like pole-preserving functions is

$$K_{v,s}(n) = \sum_{t=0}^{M-1} v_t s_{t+n} \qquad n=0,1,\ldots,N-M \tag{7}$$

Signal to Noise Ratio Improvement

The application of (6) to $\{x_t\}$ gives

$$K_x(n) = \sum_{t=0}^{M-1} x_t x_{t+n}$$
$$= \sum_{t=0}^{M-1} s_t s_{t+n} + \sum_{t=0}^{M-1} s_t \varepsilon_{t+n} + \sum_{t=0}^{M-1} \varepsilon_t s_{t+n} + \sum_{t=0}^{M-1} \varepsilon_t \varepsilon_{t+n}$$

$$K_x(n) = K_s(n) + Z(n) \qquad n = 0, 1, \ldots, N-M \tag{8}$$

$\{Z(n)\}$ for $n>0$ is a zero mean correlated noise sequence. For $n>0$ the variance of $Z(n)$ approaches 0 as $M \to \infty$. The averaging of ε_t in $Z(n)$ provides a mechanism for signal to noise ratio improvement. It has been shown by McGinn [6] that the signal to noise ratio improvement obtained by applying (6) to (2) is given approximately by

$$SNR_I = \frac{SNR_{K_s}}{SNR_x} = \frac{\sum_{n=1}^{N-M} K_s^2(n)}{(N-M)M\sigma^2[1+\sum_{i=1}^{p} a_i^2]} \tag{9}$$

For a 1st order sequence this improvement is proportional to SNR_x. With $M=N/2$, $a=.9$, $N=500$ and $SNR_x=10$, $SNR_I=9.4SNR_x$. SNR_I increases for increasing Q of the pole, for example, as $|a| \to 1$, $SNR_I \to \frac{N}{4} SNR_x$. For complex pole pairs, analysis has shown that the improvement is dependent not only on Q, as in the first order case, but also on the angle of the pole. The improvement decreases as the pole approaches the imaginary axis. For higher order sequences then, the modes with highest Q and closest proximity to the real axis are the most strongly enhanced.

Since $\{K_x(n)\}$ is an all-pole sequence corrupted by noise, the same form as (2), the signal to noise ratio may be improved further by successive application of (6) (successive autocorrelation). Limitations of this approach include: the increasingly correlated nature of the noise term, the increasing suppression of some of the modes by the zeroes and the smaller number of elements in the sequence. An example of successive autocorrelation applied to a sinusoid in noise with SNR_x of $1/10$ is shown in figure 1.

All-Pole Parameter Estimation

The above ideas can be applied to all-pole parameter estimation in the following algorithm: 1) Apply (6) to the noise corrupted sequence J times to obtain signal to noise ratio improvement, 2) Use the standard covariance method to estimate the parameters from the high signal to noise ratio sequence. Stopping criteria for step 1 are based on the limitations mentioned above and are discussed in [6].

The successive autocorrelation concept was tested by applying the above algorithm to a noise corrupted all-pole sequence with J = 0, 1 and 2 with $M=N/2$. The least squares combination of the HOYWE was applied to the original noise corrupted sequences for comparative purposes. The sequences were generated from a sixth order all-pole model with zero mean independent Gaussian noise added. Twenty five realizations of length N=300 and $SNR_x=5$ were processed by both algorithms. The results were reduced to percentage rms error in frequency and radius for each pole pair as shown in table I. The pole plots are shown in figures 2 and 3.

Table I: Variance of Pole Estimates

Processing Method*	RMS Error %			
	freq1	rad1	freq2	rad2
A0	162.15	14.49	17.04	10.60
A1	.48	.11	.72	1.35
A2	.57	.10	.28	.57
B	.98	.33	.70	1.30

* AN = Application of (6) N times, B = Least Squares combination of HOYWE

A0 is the standard covariance estimator. A1 is similar to B in that the normal equations contain fourth moments of the data. The normal equations in A2 contain eighth moments of the data sequence.

The pole-location bias in A0 is evident in figure 3a. This bias is significantly reduced in the rest of the methods. None of the methods adequately estimated the lowest Q pole. The performance of A1 and B was comparable, as expected. From table I it is clear that procedure A2 reduced the variance in the second pole-pair estimate and outperformed the HOYWE approach. As a rule, the higher Q systems benefit more from repeated correlations. The estimation of sinusoids in noise is, therefore, ideally suited to this technique.

Conclusions

Successive application of pole-preserving functions to noise corrupted all-pole sequences can increase the signal to noise ratio dramatically. Conventional all-pole parameter estimators applied to these autocorrelated sequences show a significant reduction in noise induced bias. Successive autocorrelation selectively improves the parameter estimates for poles with highest Q and closest proximity to the real axis. Sequences with these characteristics (such as sinusoids) are most effectively enhanced by the successive autocorrelation method. Since the least squares combination of the HOYWE can be interpreted as a successive autocorrelation procedure, this same data dependent performance applies. Estimates better than the least squares combination of the HOYWE can result by allowing for two or more correlations, especially for low signal to noise ratio.

References

[1] Goldberger A. S., *Econometric Theory*, New York, Wiley, 1964.

[2] Chan Y. T.,"A Parameter Estimation Approach to Estimation of Frequencies of Sinusoids", *IEEE Transactions on Acoustics Speech and Signal Processing*, Vol ASSP-29, No 2, April 1981.

[3] Kay S. M., "Noise Compensation for Autoregressive Spectral Estimates", *IEEE Transactions on Acoustics Speech and Signal Processing*, Vol ASSP-28, June 1980.

[4] Gingras D., "Estimation of the Autoregressive Parameters from Observations of a Noise Corrupted Autoregressive Time Series", *ICASSP*, Paris France, 1981.

[5] Cadzow J. A., "ARMA Modeling of Time Series", *IEEE Transactions on Pattern Analysis and Machine Intelligence*, Vol PAMI-4, No 2, March 1982.

[6] McGinn D., Masters Thesis Rice University, January 1983.

[7] Makhoul J., "Linear Prediction: A Tutorial Review", *Proceedings of the IEEE*, Vol 63, No 4, April 1975.

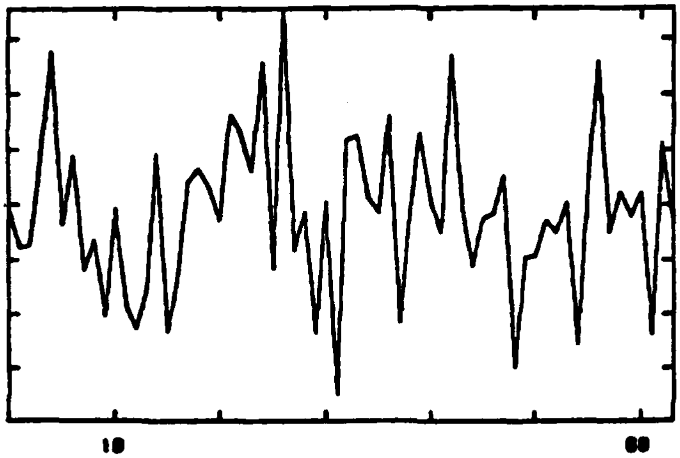


figure 1A: Sine Wave in Noise $N=500$, $SNR_x=.1$

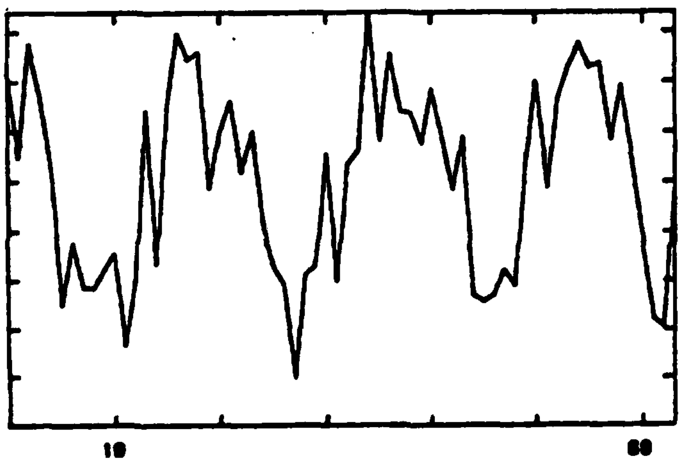


figure 1B: 1 Correlation $M=\frac{N}{2}$, $SNR_I=11.4$

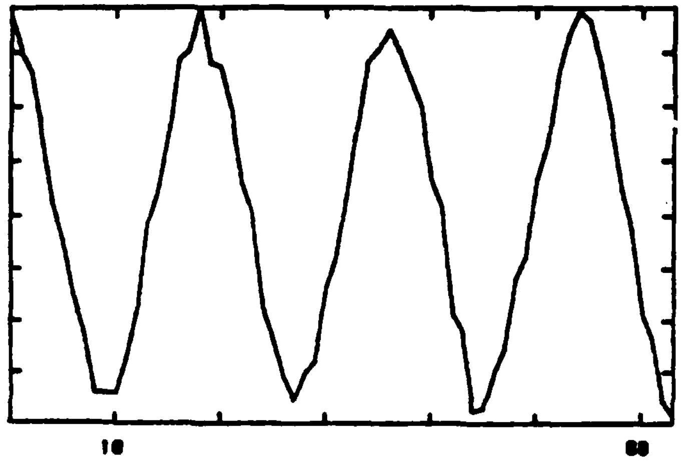


figure 1C: 2 correlations $SNR_I=486$

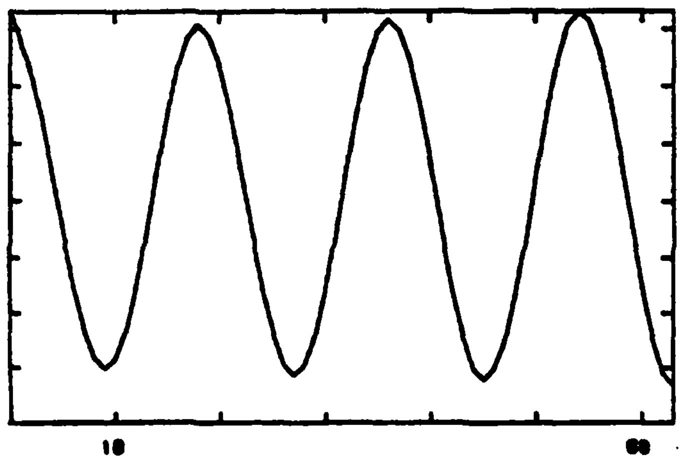


figure 1D: 3 Correlations $SNR_I=1.5\times10^5$

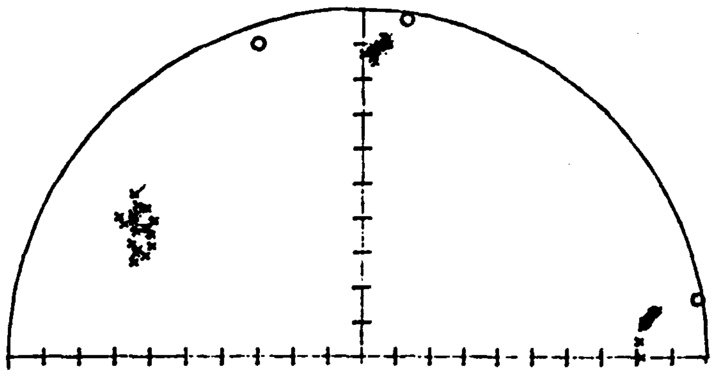


figure 2A: Covariance Method (A0)

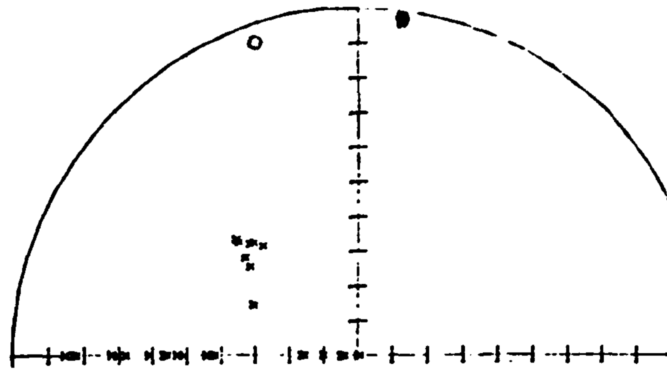


figure 2C: 2 Correlations + Covariance (

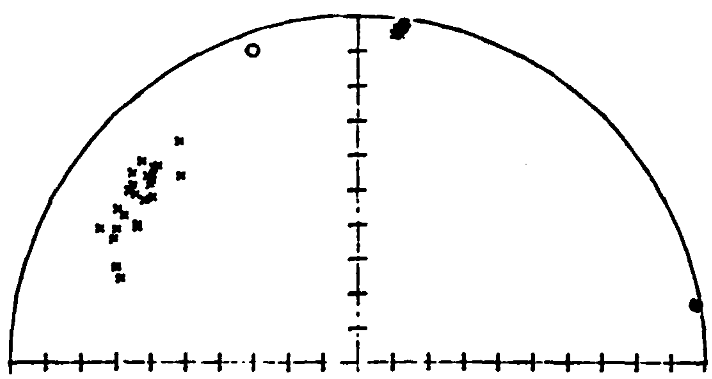


figure 2B: 1 Correlation + Covariance (A1)

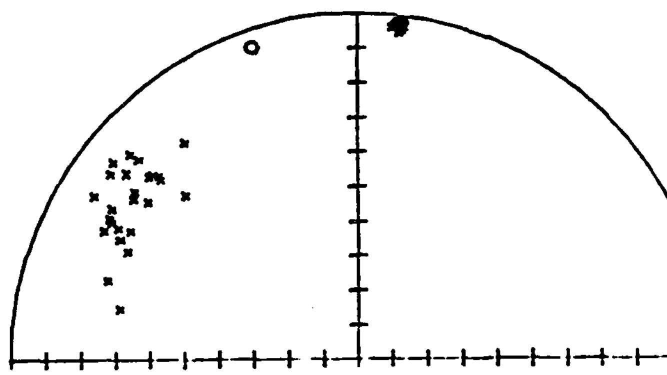


figure 3: Least Squares Comb. of HOYW

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER 8221	2. GOVT ACCESSION NO. AD-A124	3. RECIPIENT'S CATALOG NUMBER 830
4. TITLE (and Subtitle) REDUCTION OF ALL-POLE PARAMETER ESTIMATOR BIAS BY SUCCESSIVE AUTOCORRELATION		5. TYPE OF REPORT & PERIOD COVERED TECHNICAL
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) DARCY McGINN, DON H. JOHNSON		8. CONTRACT OR GRANT NUMBER(s) N00014-81-K-0565
9. PERFORMING ORGANIZATION NAME AND ADDRESS RICE UNIVERSITY P. O. BOX 1892 HOUSTON, TX 77251		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS OFFICE OF NAVAL RESEARCH ARLINGTON, VA 22217		12. REPORT DATE DECEMBER 1982
		13. NUMBER OF PAGES 10
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

APPROVED FOR PUBLIC RELEASE: DISTRIBUTION UNLIMITED.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Spectral estimation; linear prediction; all-pole models.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Conventional all-pole parameter estimators applied to noise corrupted all-pole sequences result in biased estimates. This paper describes a procedure by which reduction of this bias is accomplished by applying pole-preserving, signal to noise ratio improving functions to the sequence. Correlation like pole-preserving functions are investigated and pole dependent signal to noise ratio improvement is described. An all-pole parameter estimator using successive application of a pole-preserving function (successive autocorrelation) is given. Comparison is made with the least

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

S/N 0102-LF-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

squares combination of the higher order Yule-Walker equations, an approach to bias reduction reported by Cadzow. Successive autocorrelation is found to result in improved performance, with estimates of higher Q poles being most effectively enhanced.

END

FILMED

3-83